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13. ABSTRACT (Maximum 200 words)

This STIR work presents an exploratory investigation of helicopter stability during unsteady maneuvers on the basis of the finite time Lyapunov exponents (FTLE). These maneuvers represent short-duration dynamics that lasts long enough to stall the rotor but not long enough to reach a steady state. They also represent aggressive operations at the extremes of the flight envelope that often represent a design condition. The Floquet approach is not applicable because it requires a periodic orbit, nor is the Lyapunov-exponent approach, which requires long-time response histories. (The Lyapunov exponent reduces to FTLE under asymptotic conditions and to the real part of the Floquet exponent for a periodic orbit.) Since these aggressive maneuvers represent unsteady dynamics of short duration, the formulation exploits the largest FTLE to calculate the stability of the least stable mode from experimentally or numerically generated response data. It involves constructing a pseudo-state space by the method of delays, generating a series of Jacobian matrices, and then forming the product of these matrices to generate an Oseledec matrix and its eigenvalues. The ongoing research is still in a developmental stage; it represents the first attempt toward developing a framework for treating the stability of aggressive, short-duration maneuvers.

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Enclosure 1

Concepts and Methods of Helicopter Local Stability for Aggressive Maneuvers of Short Duration from Response Data Points

(STIR)

August 1, 2006 - April 30, 2007

Final Progress Report

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1. Foreword

Relatively little is known about helicopter stability during high-load-factor maneuvers under unsteady conditions (Refs. 1-3). These maneuvers represent short-duration dynamics that lasts long enough to stall the rotor but not long enough to reach a steady state. They also represent operations at the extremes of the flight envelope where the peak loading goes well beyond the steady thrust limit, dynamic stall occurs and often a design condition is reached (Refs. 2-4). (Routine transient maneuvers, which do not last long enough to stall the rotor as well as steady turns, which can be treated on the basis of Floquet theory, are not addressed here.)

Presently, helicopter stability is computed from state-space models and from numerically or experimentally generated responses by a host of algorithms, ranging from the moving-block analysis to sparse-time-domain technique to generalized fast-Floquet theory (Refs. 5, 6). No matter how these algorithms are designated, it is Floquet theory that underlies all of them (Refs. 5, 6). Floquet theory is a linear stability theory, and it provides asymptotic stability under periodic-response conditions. Aggressive maneuvers of short duration do not reach a steady state. What about stability on the basis of Lyapunov exponents, which, after all, is a generalization of linear stability based on Floquet theory? For example, the Lyapunov exponent reduces to the real part of the Floquet exponent for a periodic orbit and provides a means of quantifying stability of steady-state response that need not necessarily be periodic (e.g. quasi-periodic orbit, Refs. 1, 2, 7). The Lyapunov exponent, however, requires averaging of response data over a long time. Simply put, neither the

Lyapunov exponent approach nor the Floquet approach a fortiori provides a means of treating stability of aggressive maneuvers of short duration

The task of treating stability of aggressive maneuvers of short duration moves into an uncharted territory of nonlinear stability under unsteady conditions. The STIR work does just that. Specifically, it explores how the emerging finite-time Lyapunov exponents (FTLE) provide the key to determine stability and thus to provide a framework to calculate stability (Refs. 2, 7-9). This work does not address chaos; specifically, it addresses how the largest FTLE can be exploited to quantify the stability margin of the least stable mode during such maneuvers.

To help appreciate the relevance of the FTLE approach, this report begins with a passing reference to the RAH-66 Comanche helicopter lesson and to maneuvers under steady and unsteady conditions. Then it comes to FTLE as a stability measure for aggressive maneuvers of short-duration. After presenting conclusions, it also mentions the continuing study of the FTLE approach, including its application to a comprehensive set of numerically and experimentally generated data; such a continuing study should bring out the strengths and weaknesses of this approach.

2. Aggressive Maneuvers

The US Army has been emphasizing the development of air vehicles with increasingly stringent maneuvering requirements for their combat missions such as scout-attack (Ref. 4). These maneuvers are typically aggressive maneuvers, mostly of short duration, and often they represent a design condition.

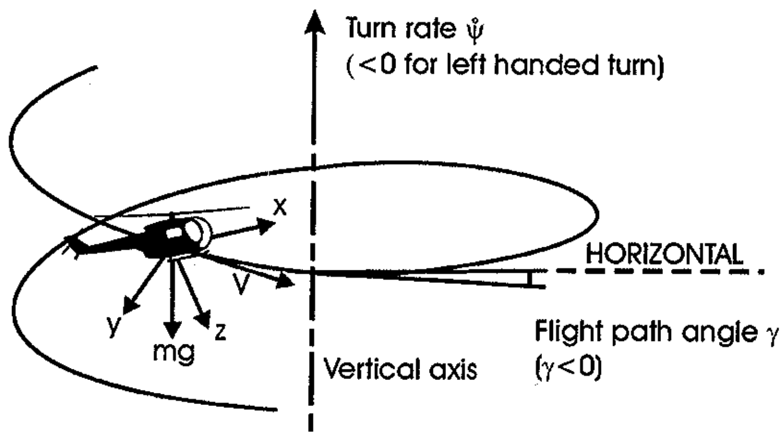

Fig. 1 Helicopter Schematic in a Diving Bank-Angle Turn.

Of particular significance is the lesson from the RAH-66 Comanche helicopter development. This vehicle was found to reach a steady condition during aggressive diving turns (Fig. 1); also, see next section for details on steady and unsteady conditions. Although this STIR work addresses unsteady conditions, the stability issue that plagued the Comanche development merits some elaboration.

Therein, inadequate stability margin or blade damping was discovered during flight tests of diving turns. Specifically, the vehicle was found to be vulnerable to air resonance instability in which the blade regressing lag mode (RLM) frequency coalesced with the fuselage roll mode frequency, and in turn the blade inplane damping sharply declined with increasing thrust level and was later augmented by an active feedback controller (Ref. 10). The state-of-the-art prediction codes failed to predict the decline; in fact, conventional wisdom has it that stability is not an issue during maneuvers. This failure demonstrated that stability during aggressive maneuvers is a crucially important and yet unresolved issue in the Army's development of air vehicles for combat missions.

3. Steady and Unsteady Conditions

During forward flight, it is not difficult to distinguish between steady and unsteady conditions and in turn to conceptualize perturbation from a periodic orbit as required by Floquet theory. By contrast, “the distinction between the steady and unsteady maneuvers is sometimes difficult to make” (Ref. 4); after all, maneuvers do involve some form of accelerated flight.

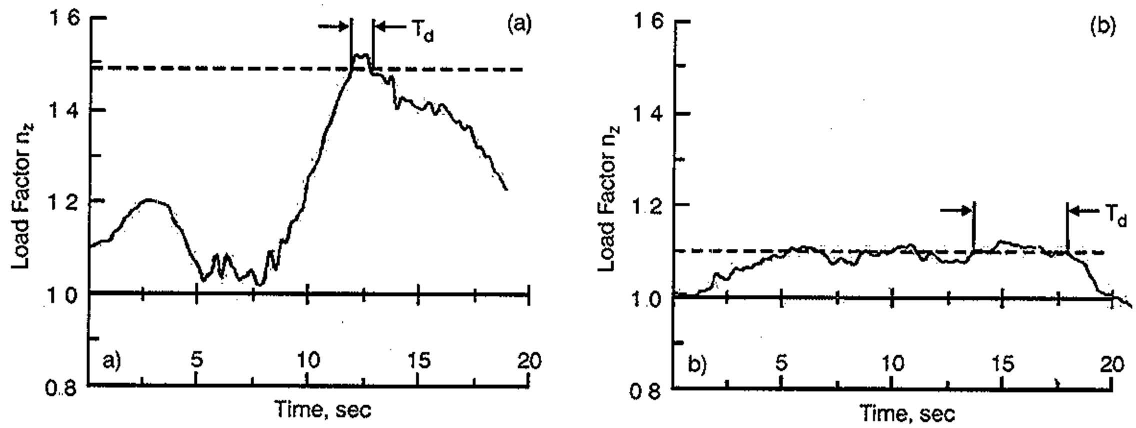

Fig. 2. “Load factor time histories in two level bank-angle turns, illustrating maneuvers steadiness Duration time, Td, indicates portion of time history where load factor is within 2% of maximum value. a) Counter 8539; b) Counter 8826.”

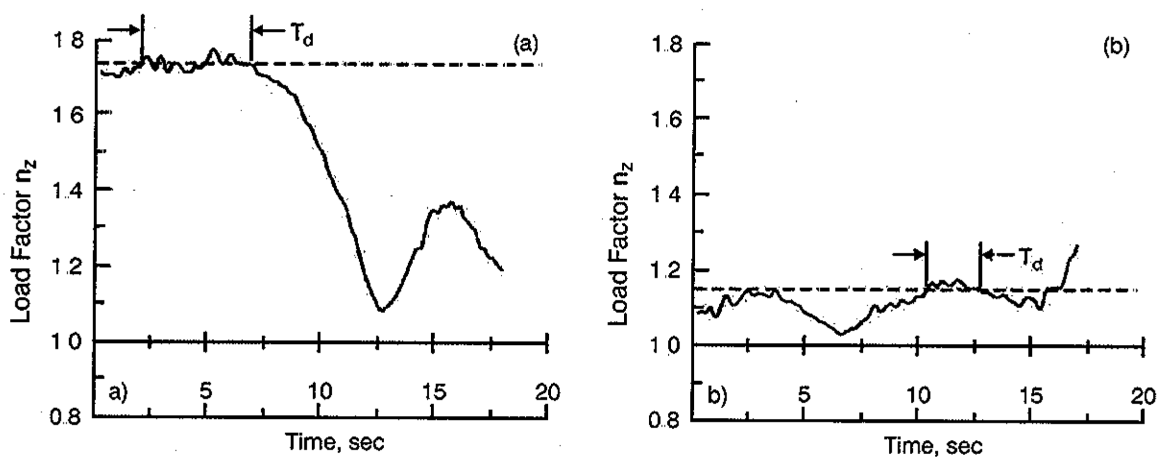

Fig. 3. “Load factor time histories in two diving bank-angle turns, illustrating maneuvers steadiness. Duration time, Td, indicates portion of time history where load factor is within 2% of maximum value. a) Counter 11668; b) Counter 11683.”

For a better appreciation of this subtle distinction, it is instructive to study Figs. 2 and 3, which show load-factor time histories for the level bank-angle and diving bank-angle turns from the UH-60A airloads program (Ref. 4). As seen from Fig. 2a (counter 8539), the maximum load factor n_z is 1.52g and the horizontal dotted line is 98% of the maximum or 1.49g. The maneuver point duration T_d is defined for the period during which the load factor n_z exceeds the 98% level, that is, within 2% of maximum value; this duration is taken as a measure of the steadiness of the maneuver (Ref. 4). While the maneuver point duration $T_d = 1.1$ seconds in Fig. 2a, $T_d = 4.2$ seconds in Fig. 2b, about 4 times longer. Qualitatively, the same type of variation is depicted by the diving-turn case in Fig.3a (counter 11668) and in Fig. 3b (counter 11683). Although the identification of T_d and the corresponding measure of maneuver steadiness are arbitrary, Figs. 2 and 3 do show that these maneuvers basically fall into two categories: In the first, the maneuvers do exhibit steady conditions; that is, the maneuver point duration T_d contains multiple consecutive rotor revolutions, which can be considered a steady (periodic) condition from Floquet theory stand point. In the second, such a steady condition is not exhibited, and the Floquet theory is no longer applicable or its applicability is nebulous at best.

4. Background to FTLE

The Lyapunov exponent is the most fundamental measure or metric of stability. Its computation requires time averaging of the response vector over a long time (Refs. 7-9). In particular, it reduces to the real part of the Floquet exponent for the special case of a periodic orbit. The theoretical basis, computation and application of FTLE represent an emerging and active area of research (Refs. 9, 11, 12). FTLE is the finite-time version of the

Lyapunov exponent, and it provides a rigorous means of quantifying the finite-time or local stability for that duration of the orbit and for that initial state In the next section, stability formulation based on FTLE is presented with a passing reference to the Lyapunov exponent, in particular, to the connection between FTLE and the Lyapunov exponent. As a background to FTLE, however, a brief account of the widely used generalized Floquet approach is included in the sequel (Ref. 5). Like the generalized Floquet approach, the FTLE formulation is not restricted to state-space modeling

4.1 Generalized Floquet Approach

Classical Floquet theory applies to state-space models. For a model, say with M states, it is required to perturb or excite each of the M states about a periodic response, one state at a time, to generate the M x M perturbation matrix P at some starting time, say at $t = t_0$. It also requires that each of the corresponding M responses are measured at the end of one period, say at $t = t_0 + 2\pi$, to generate the M x M response matrix R The Floquet transition matrix (FTM) connects P and R:

$$[R]_{M \times M} = [FTM]_{M \times M} \, [P]_{M \times M} \qquad \text{---------(1)}$$

The generalized Floquet theory provides an analytical basis to predict the stability of models without a state-space representation and with numerically or experimentally generated response histories. Algorithmically, it also connects P and R, but it permits perturbation of an arbitrary number of states and generalizes the above equation (Refs. 1, 2, 5):

$$[R]_{I \times J} = [FTM]_{I \times I} \, [P]_{I \times J} \qquad \text{---------(2)}$$

The number of rows I and the number of columns J depend upon the number of states perturbed, the number of additional periods and time delays used in generating the time-shifted or pseudo states such as $x_i(t+\Delta t)$ and $x_i(t+2\Delta t)$, where $x_i(t)$ is a state and Δt is the time delay or time shift. Now, the FTM is generated by the singular value decomposition of P and R, and then by P*, the generalized inverse of P. That is, the FTM is approximated in the least-squares sense. Stated explicitly,

$$[FTM]=[R][P]^* \qquad \text{---------(3)}$$

5. FTLE Approach

While the Floquet exponents describes the stability of a periodic response, the infinite-time or global Lyapunov exponent is a generalization of the Floquet exponent in that it describes the stability of an orbit that is not constrained to be periodic. Similarly, FTLE describes stability of an orbit that does not last long enough to reach a steady state, and thus it provides the basis for the framework to calculate the stability. This framework involves three sequential computational blocks: i) constructing a pseudo-state space; ii) generating a series of partial derivative or Jacobian matrices and in turn forming the positive symmetric Oseledec matrix, and iii) diagonalizing this Oseledec matrix by the recursive QR decomposition (Ref. 9)

5.1 Pseudo-state Space

The pseudo-state space construction begins with N-point time series of the response: $\{x_1, x_2, x_3 \ldots x_i \ldots x_N\}$, where x_i is a scalar measurement at discrete time $t = t_i$. Now it is required to introduce four important parameters:

Δt = data sampling interval (assumed constant) = $t_{i+1} - t_i$

m = embedding dimension

τ = time lag = (integer) Δt

J = delay expressed in number of data points = $\tau / \Delta t$

With these ingredients, an $M \times m$ matrix [X] with M vectors is generated:

$$
\begin{aligned}
X_1 &= \left[x_1, x_{1+J}, \ldots, x_{1+(m-1)J}\right] \\
X_J &= \left[x_j, x_{2J}, \ldots, x_{mJ}\right] \\
X_M &= \left[x_M, x_{M+J}, \ldots, x_{M+(m-1)J}\right]
\end{aligned}
$$

where X_i is the reconstructed state of the system at $t = t_i$

It is verified that N = M + (m-1) J The local Lyapunov exponents are computed in the embedded space of these M vectors

5.2 Toward Computing FTLE

Say m_o Lyapunov exponents are computed. With k representing the starting time step $t = t_i = k$, the stipulation is that the response vector X(k) is governed by the mapping (Refs. 2, 8, 9):

$$X(k) \rightarrow X(k+1) = F(X(k))$$

Roughly speaking, stability shows how perturbation $\Delta X(k)$ to X(k) grows or decays with time. Accordingly, to connect $\Delta(k)$ with $\Delta(k+1)$, set

$$
\begin{aligned}
X(k+1) + \Delta(k+1) &= F(X(k) + \Delta(k)) \\
&\approx F(X(k)) + DF(X(k)) \cdot \Delta(k)
\end{aligned}
$$

In other words,

$$\Delta(k+1) = DF(X(k)) \cdot \Delta(k)$$

where DF(X(k)) represents the $m_o \times m_o$ Jacobian or partial derivative matrix such that

$$DF(X)_{ij} = \frac{\partial F_i(X)}{\partial X_j}; \quad i = 1, 2, \ldots, m_o, \text{ and } j = 1,2, \ldots, m_o$$

It is instructive to observe that when $X(k+2\pi) = X(k)$, the stability is governed by Floquet theory. After L time steps, it is verified that

$$\begin{aligned} \Delta(k+L) &= DF(X(k+L-1)) \cdot DF(X(k+L-2)) \ldots DF(X(k)) \cdot \Delta(k) \\ &= DF^L(X(k)) \; \Delta(k) \end{aligned} \qquad \text{---------(4)}$$

By definition, the positive symmetric Oseledec matrix after L time steps is given by

$$OS(X,L) = \left[DF^L(X)\right] \cdot \left[DF^L(X)\right]^T \qquad \text{---------(5)}$$

The matrix has real eigenvalues; their logarithms are called the finite-time Lyapunov exponents and are represented by λ_i (x, L), λ_1 (x, L) being the largest. Although the Oseledec matrix is ill-conditioned, the recursive QR decomposition provides a means of computing the eigenvalues. It is instructive to observe that

$$\lim_{L \to \infty} \lambda_i(x,L) = \lambda_i, \quad \lambda_1 > \lambda_2 > \cdots > \lambda_{m_o} \qquad \text{---------(6)}$$

where the logarithm of λ_i is the conventional Lyapunov exponent. For regular motion, $\lambda_1 \leq 0$ and for chaos, $\lambda_1 > 0$. Hereafter, the treatment is restricted to stable regular motion and to the largest FTLE, which is simply designated as λ_{1L} (< 0)

5.3 FTLE as a Stability Measure

Stability means the exponential rate of growth or decay with time of two neighboring trajectories; this is shown schematically in Fig. 4. Therein, δ (t) represents the separation between the trajectory x (t) with initial state x_o and the perturbed trajectory x (t) + δ (t) with initial state $x_o + \delta_o$. After L time steps, the separation δ (t) can be represented as

$$\|\delta(t)\| \sim \|\delta_o\| e^{\lambda_{1L} t} \qquad \text{---------(7)}$$

With $L \rightarrow \infty$, $\lambda_{1L} \rightarrow \lambda_1$, and for a periodic trajectory with period T,

$$\lambda_1 = \frac{1}{T} \ln|\mu_1| \qquad \text{---------(8)}$$

where μ_1 is the largest characteristic multiplier or the largest eigenvalue of the FTM, given by Eq. (3). For a stable system, it is instructive to observe that $-\lambda_1$ (> 0) provides the stability margin of the least stable mode.

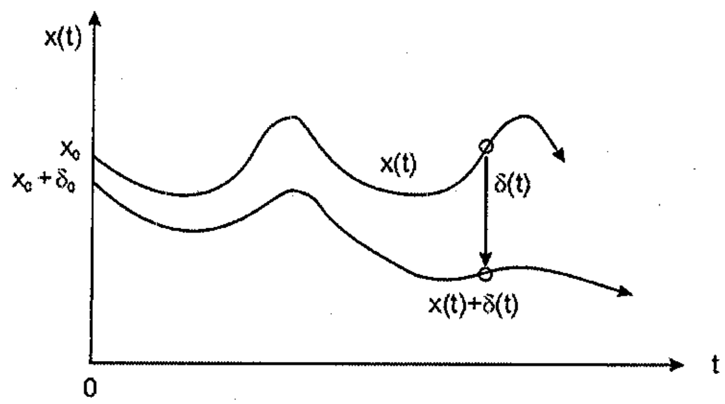

Fig. 4 Schematic of Separation between two Neighboring Trajectories

For rotorcraft systems designed for aggressive maneuvers, the stability issue often centers on how stable is the least stable mode, that is, on the adequacy of the stability margin from handling qualities perspective. Thus, what is required is a means of quantifying the stability margin. Recall that $-\lambda_1$ (> 0) quantifies the stability margin for the least stable mode; it is evaluated from Floquet theory for a periodic trajectory and from Lyapunov-exponent method for a trajectory that is not constrained to be periodic. Furthermore, $\lambda_{1L} \rightarrow \lambda_1$ with $L \rightarrow \infty$;

that is, λ_{1L} serves as the finite-time version of λ_1. All this points to the feasibility of using $-\lambda_{1L}$ (> 0) as a sensible means of quantifying the stability margin of the least stable mode for aggressive maneuvers of short duration.

6. Conclusions

The investigation toward developing a framework to calculate helicopter stability during aggressive, short-duration maneuvers has been presented. This investigation exploits the largest finite time Lyapunov exponent as a measure of stability margin, and it represents the first attempt toward such a development from first principles. It is still in an exploratory stage. The next section gives an account of its continuation under a three-year ARO grant (May 1, 2007 - April 30, 2010).

7. Continuing Study

Two major objectives of the continuing study are 1) to further refine the analytical basis of the developed framework to calculate the stability and 2) to investigate the computational issues.

The computation of the largest finite time Lyapunov exponent λ_{1L} represents an active research area and is computationally demanding. One reason is that the Oseledec matrix given by Eq. (5) is ill-conditioned. Accordingly, the recursive QR decomposition will be investigated to reliably compute λ_{1L} (Ref. 9).

Finally, the database from the UH-60 airloads program will be used to identify aggressive maneuvers of short duration and then to calculate the stability margins for these maneuvers. This should bring out the strengths and weaknesses of the framework to calculate the stability of aggressive, short-duration maneuvers on the basis of the largest finite-time Lyapunov exponent.

8. References

1. Goankar, G. H., "Helicopter Stability from Floquet Theory and a Preliminary Study of Nonlinear Aspects," Proceedings of the 9th International Workshop on Aeroelasticity of Rotorcraft Systems, co-sponsored by the Army Research Office and the University of Michigan, Ann Arbor, Michigan, October-22-24, 2001.
2. Gaonkar, G. H., " Concept and Methods of helicopter Local Stability for Aggressive Maneuvers of Short Duration from Response Data Points," Proceedings of the 11th International Workshop on Rotorcraft Dynamics and Aeroelasticity, co-sponsored by the Army Research office and Florida Atlantic University, Boca Raton, Florida, October 12-14, 2005.
3. Omiston, R. A., and Preston, P. B., "Rotor Blade Stall Flutter, stable or Unstable? An Exploratory Investigation of Nonlinear Aeroelasticity," Proceedings of the American Helicopter Society 63rd Annual Forum and Technology Display, Virginia Beach, Virginia, May 1-3, 2007, pp 1322-1337.
4. Boushman, W. G., and Kufeld, R. M., UH-60A Airloads Catalog, NASA TM 2005-212827, August 2005.

5. Gaonkar, G. H., "Serial and Parallel Computing of Helicopter Stability from Floquet and Approximate Methods," Paper presented at the 41st AIAA/ASME/ASCE/AHS/ASC structures, Structural Dynamics, and Materials Conference, April 3-6, 2000, Atlanta, GA, AIAA 2000-1692.

6. Bauchau, O. A., and Wang, Jielong, "Efficient and Robust Approaches for Rotorcraft Stability Analysis," Proceedings of the American Helicopter Society 63rd Annual Forum and Technology Display, Virginia Beach, Virginia, May 1-3, 2007, pp. 1307-1321.

7. Parkar, T. S., and Chua, L. O. Practical Algorithms for Chaotic Systems, Springer Verlag, New York, 1989, Chapter 3.

8. Nayfeh, A. and Balachandran, B. Applied Nonlinear Dynamics, John Wiley and Sons, Inc. New York, 1995, Chapter 7.

9. Abarbanel, H. D. I., Analysis of observed Chaotic Data, Springer Verlag, New York, 1996, Chapter 5.

10. Panda, B., Mychalowycz, E., Kothmann, B. and Blackwell, R., "Active Controllers for Comanche Air Resonance Stability Augmentation," Proceedings of the American Helicopter Society 60th Annual Forum, Baltimore, Maryland, June 7-10, 2004.

11. Ott, Edward, Chaos in Dynamical Systems, Cambridge University Press, New York, Second Edition, 2002, Chapter 9.

12. Shadden, S. C., A Dynamical Systems Approach to Unsteady Systems, doctor of philosophy thesis, California Institute of Technology, April 2006, Chapters 1 and 2.